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Problem set on Constraint optimization

Q-1 Use the method of lagrange multipliers in the following problems.

(a) Maximize x^2y on the unit circle

(b) Find all points on the hyperbola $xy=1$ that lie closest to the origin

(c) Find all points on the surface $xyz=1$ closest to the origin.

(d) Minimize $f(x,y,z)=x^2+y^2+z^2$ on the line of intersection of the planes $x+2y+3z=0$ and $2x+3y+z=-4$

(e) The plane $2y+4z-5=0$ meets the cone $z^2=4(x^2+y^2)$ in a single curve. Find the point on this curve closest to the origin.

(f) Maximize$_{(x,y)}$ x^2+y^2 st $x^2+xy+y^2=3$

Q-2 A firm that uses two inputs to produce output has the production function $4x^{1/4}y^{3/4}$. The price of output is 1 and the price of each input is 1. The firm is constraint to use exactly 1000 units of input x.

(a) How much of input y does it use?

(b) What is approximately the maximum amount the firm is willing to pay to be allowed to use ε more units of input x, for ε small?

Q-3 The output of a good is $x^a y$ where x and y are the amounts of two inputs and $a > 1$ is a parameter. A government controlled firm is directed to maximize output subject to meeting the constraint $2x + y = 12$.

(a) Solve the firm's problem.

(b) Use the envelope theorem to find how the maximal output changes as the parameter a varies.