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CYCLICAL POSSIBILITIES IN A MODEL OF A MONOPOLISTICALLY COMPETITIVE INDUSTRY

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ABSTRACT

A dynamic model of monopolistic competitive industry is set out which is based on standard textbook formulations of a linear demand function and cubic cost function. The existence of the equilibrium, local stability and comparative dynamic properties of the model are set out. The Hopf Bifurcation theorem is employed to establish the possibility of cycles in the model.

KEYWORDS: Monopolistic Competition, Hopf Bifurcation, Output, Number of Firms.

INTRODUCTION

A model of monopolistic competition was set out in the works of Chamberlin (1969) and Robinson (1969)[1]. Textbooks such as Corchón (2001) provide only a verbal (or graphical) discussion of the model of monopolistic competition put forward in Chamberlin (1969). A large literature has developed regarding monopolistic competition drawing on the work of Dixit and Stiglitz (1977) but it often does not engage with the themes highlighted by Chamberlin (1969) in an elementary manner.

In this paper a simple mathematical model of monopolistic competition drawing on the work of Henderson and Quandt (1980) is set out where standard textbook assumptions of a linear inverse demand function and a cubic cost function hold good. However this model may give rise to cyclical trajectories when formulated dynamically. In section 2 a simple model of monopolistic competition drawing on the work of Henderson and Quandt (1980) is set out; in section 3 a dynamic reformulation of the model is undertaken and its local stability is examined; in section 4 a comparative static exercise for the model is undertaken; in section 5 the possibility of cyclical trajectories in the model is explored; and section 6 concludes the paper.

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1 The differences in the formulations of Chamberlin (1969) and Robinson (1969) are not germane to this paper.

A Simple Model of Monopolistic Competition

Henderson and Quandt (1980) present a simple model of monopolistic competition which forms the staring point of the model of this paper. The inverse demand function of a firm in an industry which is characterised by monopolistic competition is linear and decreasing function of its own output (q). It is a linear and decreasing function of the outputs of all other firms (Q). The output of each firm is differentiated from that of its rivals but this is not manifested in the inverse demand function. There are n firms in the industry and this number varies over time.

$$p = -\beta(n-1)Q - \beta q + \alpha \qquad \text{.........................(1)}$$

It is assumed that every firm ignores the actions of other firms and any change in the number of firms that occurs over time when it chooses its level of output.

The total cost of the firm is a cubic function of output. Since this model is concerned with the long run there are no fixed costs.

$$C = aq^3 - bq^2 + cq \qquad \text{.........................(2)}$$

Vali (2014) provides the following restrictions on the parameters of this cost function[2] for it to be economically meaningful:

1. The level of output must be positive when marginal cost is at its minimum. This results in the condition:

$$\frac{b}{3a} \succ 0 \qquad \text{.........................(3)}$$

2. For the marginal cost function to have an authentic minimum the second derivative of the marginal cost function must be positive. This results in the condition:

$$a \succ 0 \qquad \text{.........................(4)}$$

From equations (3) and (4) and it follows that:

$$b \succ 0 \qquad \text{.........................(5)}$$

3. Marginal cost must be positive when it is at its minimum. This results in the condition:

$$b^2 \prec 3ac \qquad \text{.........................(6)}$$

Due to equations (4) and (6) it follows that:

$$c \succ 0 \qquad \text{.........................(7)}$$

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2 Vali (2014) uses a cubic cost function whose quadratic term has an undetermined sign while in the current paper b is assumed to be positive. Due to equation 5 the two are equivalent.

The expression for the profit of the firm is:

$$\pi = q(-\beta(n-1)Q - \beta q + \alpha) - (aq^3 - bq^2 + cq) \qquad \text{........................(8)}$$

The first order condition for the profit of the firm to be a maximum is:

$$\frac{d\pi}{dq} = -\beta(n-1)Q - aq^2 - q(2aq - b) + bq - 2\beta q - c + \alpha = 0 \qquad \text{........................(9)}$$

Since it is assumed that all firms in the industry are identical it follows that:

$$q = Q \qquad \text{......................(10)}$$

Due to equation (10), equation (9) may be expressed as:

$$-aq^2 - q(2aq - b) - \beta(n-1)q + bq - 2\beta q - c + \alpha = 0 \qquad \text{......................(11)}$$

The second order condition for the profit of the firm to be a maximum is:

$$-6aq + 2b - 2\beta \prec 0 \qquad \text{......................(12)}$$

Entry and exit of firms will cease when the level of profit of each firm in the industry is zero:

$$q(-\beta(n-1)Q - \beta q + \alpha) - (aq^3 - bq^2 + cq) = 0 \qquad \text{......................(13)}$$

Equations (11) and (13) (and letting $Q = q$) may be jointly solved for the equilibrium levels of output (q) and number of firms (n):

$$q = \frac{b-\beta}{2a} \qquad \text{......................(14)}$$

$$n = -\frac{\beta^2 - 4\alpha a - b^2 + 4ac}{2\beta b - 2\beta^2} \qquad \text{......................(15)}$$

For the equilibrium to be viable it must be the case that both the following inequalities must be valid:

$$b - \beta \succ 0 \qquad \text{......................(16)}$$

$$\beta^2 - 4\alpha a - b^2 + 4ac \prec 0 \qquad \text{......................(17)}$$

Equation (16) is equivalent to equation (12) to when the latter is evaluated at the equilibrium level of output. Further it may be established that at the equilibrium level of output average cost exceeds marginal cost:

$$AC(q) = \frac{\beta^2 - b^2 + 4ac}{4a} \qquad \text{......................(18)}$$

$$MC(q) = \frac{3\beta^2 - 2\beta b - b^2 + 4ac}{4a} \qquad \text{......................(19)}$$

$$AC(q) - MC(q \quad) = \frac{\beta(b-\beta)}{2a} \succ 0 \qquad \text{......................(20)}$$

A Dynamic Formulation of the Model

It is possible to reformulate the first order condition and the zero profit condition as dynamic processes. But in order to do so the number of firms (n), which can only take on integer values, must be replaced by another variable which can vary continuously namely the output of the industry (ω) where:

$$\omega = nq \qquad \text{.......................(21)}$$

A dynamic process of output adjustment by the firm in the industry is one where output is increased when marginal profit ($d\,\pi/d\,q$) is positive and vice versa:

$$\frac{dq}{dt} = \gamma(-aq^2 - q(2aq - b) + bq - 2\beta q - \beta(\omega - q) - c + \alpha) \qquad \text{.......................(22)}$$

Here γ is the speed of output adjustment by the firm. In equation (22) , the symbol for output of other firms (Q) has been replaced by q for reasons of brevity.

In terms of q and ω, equation (13) may be expressed as a dynamic process as follows:

$$\frac{dq}{dt} = f(q, n) \qquad \text{.......................(23)}$$

Here $f(q, n)$is given by the following expression:

$$\eta\, q\left(q\left(-\beta\left(\omega - Q\right) - \beta\, q + \alpha\right) - q\left(c - b\, q + a\, q^2\right)\right) + \frac{\gamma\,\omega\left(\alpha - c - \beta\left(\omega - q\right) - 2\,\beta\, q + b\, q - q\left(2\, a\, q - b\right) - a\, q^2\right)}{q}$$

Here the number of firms rises when positive profits are made and vice versa and η is the speed of firm entry (exit) into (from) the industry.

Equations (22) and (23) form a two dimensional dynamical system. The conditions for local stability of this dynamical system may be expressed in terms of the resulting characteristic equation (Gandolfo (1980)):

$$b_1 \succ 0 \qquad \text{.......................(24)}$$

$$b_2 \succ 0 \qquad \text{.......................(25)}$$

$$\lambda^2 + b_1\lambda + b_2 = 0 \qquad \text{.......................(26)}$$

In terms of the model consisting of equations (22) and (23) , the local stability conditions (equations (24) and (25)) may be expressed as:

The following expression (equation 27) must be negative:

$$\frac{\beta^4\,\eta - 6\,\gamma\,\beta^2\, a^2 - 8\,\gamma\,\alpha\, a^3 - 3\,\beta^3\,\eta\, b + 12\,\gamma\,\beta\, a^2\, b + 3\,\beta^2\,\eta\, b^2 - 6\,\gamma\, a^2\, b^2 - \beta\,\eta\, b^3 + 8\,\gamma\, a^3\, c}{4\, a^2\left(b - \beta\right)} \qquad \text{...(27)}$$

$$\frac{\gamma\eta(b-\beta)^3\beta}{4a^2} \succ 0 \qquad \text{.......................(28)}$$

Equation (16) ensures that equation (28) is satisfied. Equation (27) will be satisfied if:

$$\eta \prec \frac{2\gamma a^2(-3\beta^2-4\alpha a+6\beta b-3b^2+4ac)}{\beta(b-\beta)^3} \qquad \text{.......................(29)}$$

The satisfaction of equation (29) requires:

$$-3\beta^2 - 4\alpha a + 6\beta b - 3b^2 + 4ac \succ 0 \qquad \text{.......................(30)}$$

Equations (17) and (30) together imply:

$$b^2 - \beta^2 \succ 4a(c-\alpha) \succ 3(b-\beta)^2 \qquad \text{.......................(31)}$$

In turn equation (31) implies that:

$$2\beta \succ b \qquad \text{.......................(32)}$$

$$c \succ \alpha \qquad \text{.......................(33)}$$

An economic description of local stability could proceed as follows: let a small parametric change (an increase in α for instance) cause the industry to veer of its previous equilibrium. Both the output of the firm and the output of the industry will rise as is evident from an inspection of equations (22) and (23). This will have contradictory effects on the dynamic trajectory of the output of the firm and the industry. If the above mentioned local stability conditions hold good then the industry will converge to its new equilibrium.

Thus the speed of entry (exit) of firms into (from) the industry (η) must be below a critical level for the dynamic trajectory of the industry to be locally stable, the quadratic coefficient of the cost function (b) must be higher than the slope of the inverse demand function (β) but not too high. Further the linear coefficient of the cost function (c) must exceed the intercept term of the inverse demand function (α). If the latter two conditions are not met then the upper bound on η will be negative which will mean that local stability is not possible.

Comparative Statics

The comparative static properties of the model may be evaluated. To begin with equations (23) and (24) may be solved for q and ω[3]:

$$q = \frac{b-\beta}{2a} \qquad \text{.......................(34)}$$

$$\omega = -\frac{\beta^2-4\alpha a-b^2+4ac}{4\beta a} \qquad \text{.......................(35)}$$

3 The non-viable equilibrium with non-positive values of the variables is not considered.

The conditions for a positive value of industry output is not examined here since if the firm output and the number of firms are both positive (and discussed in section 2) then industry output will be positive.

In all there are five parameters α, β, a, b and c. The impact of a change in all five parameters on the equilibrium are examined seriatim.

A change in α will leave equilibrium firm output unchanged while the output of the industry will rise:

$$\frac{dq}{d\alpha} = 0 \qquad \text{.......................(36)}$$

$$\frac{d\omega}{d\alpha} = \frac{1}{\beta} \succ 0 \qquad \text{.......................(37)}$$

In other words, the number of firms in the industry will rise when there is a rise in α.

A change in β will reduce equilibrium firm output and industry output:

$$\frac{dq}{d\beta} = -\frac{1}{2a} \prec 0 \qquad \text{.......................(38)}$$

$$\frac{d\omega}{d\beta} = \frac{-\beta^2 - 4\alpha a - b^2 + 4ac}{4\beta^2 a} \prec 0 \qquad \text{.......................(39)}$$

Equation (17) implies that the numerator of equation (39) is negative. In other words, the number of firms in the industry will rise when there is a rise in β.

A change in a will reduce equilibrium firm output and industry output:

$$\frac{dq}{da} = -\frac{b-\beta}{2a^2} \prec 0 \qquad \text{.......................(40)}$$

$$\frac{d\omega}{da} = -\frac{(b-\beta)(\beta+b)}{4\beta a^2} \prec 0 \qquad \text{.......................(41)}$$

The number of firms in the industry will fall when a rises as is clear from an inspection of equation (15).

An increase in b will increase equilibrium firm output and the output of the industry:

$$\frac{dq}{db} = \frac{1}{2a} \succ 0 \qquad \text{.......................(42)}$$

$$\frac{d\omega}{db} = \frac{b}{2\beta a} \succ 0 \qquad \text{.......................(43)}$$

Thus the number of firms in the industry will rise when there is a rise in b.

An increase in c will leave equilibrium firm output unchanged while the output of the industry will fall:

$$\frac{dq}{db} = 0 \qquad \text{.......................(44)}$$

$$\frac{d\omega}{db} = -\frac{1}{\beta} \prec 0 \qquad \text{.......................(45)}$$

Thus the number of firms in the industry will fall when there is a rise in c.

Cyclical Possibilities in the Industry

The possibility of the existence of cycles in continuous dynamical systems may be established by the use of the Hopf Bifurcation. Briefly a two dimensional continuous dynamical system undergoes a Hopf Bifurcation when a parameter changes and causes the characteristic equation to acquire two eigenvalues with complex conjugate values with zero real parts (Kuznetsov (2004))[4]. This gives rises to a cyclical trajectory that bifurcates from the equilibrium[5].

Flaschel (2009) provides the following statement on the existence of a Hopf Bifurcation in a two dimensional dynamical system which in terms of the model considered here may be stated as follows: the characteristic equation $\lambda^2 + b_1\lambda + b_2 = 0$ (equation (26)) will have a pair of complex conjugate roots with zero real parts if and only if it the case that $b_1 = 0$ and $b_2 \succ 0$.

The two dimensional dynamical system consisting of equations (22) and (23) undergoes a Hopf Bifurcation when the parameter η attains a critical value $\overline{\eta}$ (and therefore $b_1 = 0$):

$$\overline{\eta} = \frac{2\gamma a^2(-3\beta^2 - 4\alpha a + 6\beta b - 3b^2 + 4ac)}{\beta(b-\beta)^3} \qquad \text{.......................(46)}$$

CONCLUSION

It has been established that an elementary model of monopolistic competition when formulated dynamically could give rise to a cyclical trajectory. Future work could go on to explore:

1. The role of joint production in this elementary model of monopolistic competition.

2. The consequences of dropping the assumption that the normal rate of profit in the rest of the economy is not zero but positive.

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4 Kuznetsov (2004) points out that it is more appropriate to term the Hopf Bifurcation, the Andronov-Hopf Bifurcation.

5 The cyclical trajectory may be either attracting or repelling (Kuznetsov (2004)). The calculation of the stability of the Hopf Bifurcation is left for future work.

3. The stability of the cyclical trajectory that emerges due to the Hopf Bifurcation in the model.

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