WHAT MAKES VOLATILITY SMILE?: AN EMPIRICAL INVESTIGATION OF IMPLIED VOLATILITY FUNCTIONS IN INDIAN MARKET

Shivani Inder[1] and J. S. Pasricha[2]

The Black Scholes Option Pricing Model is one of the most widely used option pricing model and it assumes the implied volatility input is constant across different levels of moneyness and time to expiry. However, in practice it has been observed as varying across different dimensions by researchers. This non constant nature of implied volatility as function of moneyness is termed as volatility smile. The paper analyzes the one month Call options contracts on the underlying asset- Nifty from the time period June 2001 to Decemeber 2012. The Implied volatility functions used to explain the phenomenon of 'Volatility Smile' have been analyzed in order to find the an appropriate explanation for the volatility smile and its curvature.

Key words: Implied Volatility, Smile, Moneyness, Options, Black Scholes.

INTRODUCTION

The Black Scholes Option Pricing Model is considered as the cornerstone for the option pricing theory. The formula provides a unique solution or price to an option which has an underlying asset operating in an ideal, complete and unconstrained market. It considers the notion of 'absence of arbitrage opportunities' as one of its foundation principle. It means that in a complete market under proper assumptions, each contingent claim can be replicated exactly by a synthetic portfolio consisting of the risk free asset and the underlying asset. This price can be coincided with the expectation of the claim's discounted value under a new "risk neutral" equivalent martingale probability measure (Shi, 2005). The only unknown input in the Black Scholes formula is the implied volatility which is a measure of the fluctuation of the value of the underlying asset and represents the degree of risk to the option seller. Black Scholes assume that all option prices with varying exercise prices, but same underlying asset and time to expiry have same implied volatility. When the Black Scholes formula is inverted to back out

[1] Assistant Professor, Chitkara Business School, Chitkara University, Punjab

[2] Professor, Department of Commerce, Punjabi University, Punjab

volatilities from option prices, the implied volatility estimates varied across different levels of moneyness and time to expiry. The non constant nature of the volatility has been explored in the form of volatility smile/skew and has gained a lot of importance in hedging and market behaviour explanation. Considering all this, the main motivation of this paper is to examine the behaviour of the volatility in Indian options market, which is carried out by studying the pattern of volatility smile and analysis of the explanatory models for volatility smile.

REVIEW OF LITERATURE

Geske and Roll (1984) showed that both in the money and out of the money options contained volatility bias. They concluded that time and money bias might be related to improper boundary conditions whereas the volatility bias problem might be the result of the statistical errors in estimation. Rubinstein (1985) used 30 most liquid CBOE option classes from August 1976 to August 1978 and c oncluded that systematic deviations from the Black Scholes model appear to exist, but the pattern of deviation varies over time. Shastri and Wethyavivorn (1987) examined the volatility pattern implied by the options written on Japanese Yen, Swiss Franc, and West German Mark exchange rates on the floor of the Philadelphia Stock Exchange (PHLX) covering the period from March 1, 1983 to August 31, 1984. They found that mean implied volatilities are U-shaped functions of the spot exchange rate to exercise price ratio, except for options having maturity time of more than 220 days. Chewlow and Xu(1993) examined volatilities implied from option prices traded on Chicago Mercantile Standard & Poor 500 Index futures for the time period of 1985 to 1992, and showed that the implied volatilities exhibit substantial skews in addition to the smiles.

Duque and Paxson (1994) also found the smile effect for options traded on LIFFE and speculated that there is a possible empirical relationship between time to maturity and the U-shaped smile. Rubinstein (1994) analysed S&P 500 index options for the latter part of the eighties and found that the crash of 1987 appeared to have increased the market participation with view of possibility of another market crash in near future. This 'crash-o-phobia' causes a slightly bimodal implied distribution to be quite common after the crash. The pre-crash 'smile' pattern in the implied volatilities also appears to have changed into a 'sneer' in the post-crash period. Taylor and Xu (1994) suggested that stochastic volatility was the possible explanation for volatility smile. They used the data from Philadelphia Stock Exchange (PHLX) over the period from Nov 1984 to Jan 1992

Wang (2002) attempted to find out the function which characterize the regularities in the skew pattern of Black Scholes Implied Volatility and also investigated the stability of volatility functions. He employed S&P 500 call options traded on CBOE from March 1, 1996 to May 31,1996 .He found that in case of both in-sample and out-of-sample pricing the quadratic function of maturity adjusted moneyness reduced the maturity related and moneyness related pricing error to the greatest extent. Shi Q. (2005) examined the smile behaviour in HangSeng Index options market in HongKong. The results indicated that the implied volatilities for HangSeng Index Options tended to smile consistently. He also found that the volatility of underlying asset and time to expiration seem to be key variables that explained the variability of Implied volatility function.

Misra D, Kannan R, and Misra S D (2006) studied the existence of volatility surfaces in case of NSE Nifty options and found the other determinants of implied volatility. They found that deeply in the money and deeply out of the money options had higher volatility than at the money options, and the implied volatility of out of the money call options was more than in the money calls. Further the implied volatility was higher for the month contracts than for near the month contracts, whereas deeply in the money and out of the money options with shorter maturity had higher volatility than those of with longer maturity. Also, put options had higher volatility than call options, and implied volatility of more liquid options was more than that of less liquid options. The study found that the shape of the volatility smile in India was similar to that which was prevailing in US before the stock market crash of 1987.They concluded that deeply in the money and out of the money options were having higher implied volatility than at the money options which might be due to the fact that these lacked liquidity and the seller of these options demand liquidity premium to price deeply in the money and out of the money options.

Malabika Deo, K. Devanadhen and K. Srinivasan (2008) examined the implied volatility function for selected individual equity call options from Indian Stock Market for the period of one year and tried to find out U-shaped smile pattern, where the volatilities were averaged within groups according to their moneyness and potential explanatory variables of implied volatility function. They evaluated that the implied volatilities of in-the-money option were higher than implied volatility of out-of-the-money option. They fitted different specifications of volatility functions and found that a linear and quadratic function with moneyness and time to expiration contributed to the greater specification of implied volatility in equity call options to all levels of moneyness. They excluded options with less than 8 and more than 90 trading days to expiration, in order to avoid the liquidity – related biases and found that the maturity approach changed the options smile asymmetry converting a "Wry grin" typical for longer term series into a "Reverse grin" for nearly expiring options, with a more or less symmetric smile for middle term options. The implied volatilities for all the strike prices of an option at the time of maturity were surfaced approximately like U-shaped. Hence, a smile existed because the implied volatility of in-the-money and out-of-the money options was higher than the at-the-money options. Finally, when time to expiration and moneyness were combined, the explanatory power of the smile bias increased, so the investors were willing to pay a higher price for in-the-money and out-of-the-money options. Thus, higher price translated into higher implied volatility and structured like a U - Shaped smile.

Machado and Rybczynski (2011) studied the behaviour of smile in the Warsaw Stock Exchange by investigating the call and put options on the Polish WIG20 Index options in 2011.They proposed seventeen type smiles which represented all possible cases of higher,equal and lower values of implied volatility function points. Tavin (2011) obtained the risk neutral density of the underlying asset price as the function of its option implied volatility smile. Chalamandaris and Rompolis (2012) explored the role of the realized return distribution in the formation of the observed implied volatility smile for the options based on S&P 500 index. They found that the realized distribution was significant in formation of volatility smile. Singh (2013) investigated the information content of the volatility smile. He modeled the volatility smile through deterministic volatility functions of Dumas et al(1998) in the Indian market for the period of 2007-2009 and found that the deterministic volatility function approach performs better than the Black Scholes model. Tanha and Dempsey(2015) employed the structural relationships and dynamics of the volatility smile in relation to the option liquidity, key features of the underlying asset and market momentum to examine the dynamics of the volatility smile in the Australian market. They found a number of biases in the volatility resulting into characteristic smile shape.

OBJECTIVE OF THE STUDY

The main objective of the study is to analyse the shape of volatility smile with reference to Indian stock market. The study focuses on the finding the model which can explain the structure of volatility smile in Indian scenario.

DATA AND METHODOLOGY

The Study considers the one month call option contracts on the underlying asset S&P Nifty from June 2001 to June 2015. Christensen and Prabhala (1998) pointed out the problem associated with overlapping data which led to inappropriate regression results in Canina and Figlewiski (1993) work, because it caused the problem of autocorrelation between the option prices with different strikes (having variation in time to maturity) sharing same information on any two consecutive days estimate of historical volatility. They suggested the use of one month contracts for better estimation. Following on the same lines, the sampling of data points to be considered for the study is done on the basis of the following sampling plan:

Step 1: Options to be considered for the study are collected on the working day immediately following the expiry date of the previous month's options. These options approximately have one month (i.e. four to five weeks) time to expiration. This step gives us non-overlapping datapoints with maximum time to expiry of one month.

Step 2: Out of the options which have been considered for the analysis in step 1, the observation which satisfy the boundary condition i.e. $c > ((Fe^{-rt}) - X$ are selected.

Step 3: Options must have some trade volume. So the observations which had less than 50 contracts being traded on that day have been dropped. Also, observations which have the settlement price equating to zero have also been rejected out of the sample because such value would not allow the optimal solution to occur in estimation of Black Scholes implied volatility

Step 4: The options for which the optimal solution for Black Scholes implied volatility could not be converged through Newton Raphson method have also been dropped.

Step 5: The options which are finally considered for the analysis are further divided into different sub samples on the basis of moneyness and time to expiry for the analysis purposes.

The dimensions of moneyness and time to expiry are defined as follows

Moneyness: Moneyness measures the extent to which an option is in profit. It defines the relationship between the current value of the underlying index and the strike price of the option contract considered.The options, based on their moneyness i.e. $\eta = S_t / X_t$ where S_t is the index level and X_t is the exercise price of the option at time 't', are categorized into following categories

DOTM: Deep Out of The Money Options: If $\eta \in (0,0.85)$;OTM: Out of The Money Options : If $\eta \in (0.85, 0.95)$;ATM: At the Money Options: If $\eta \in (0.95, 1.05)$;ITM: In The Money Options: If $\eta \in (1.05,1.15)$;DITM: Deep In The Money Options: If $\eta \in (1.15,\infty)$

Time To Expiry (τ) : It represents the deadline of an option contract. It is equal to the time remaining for the holder of the option till he has the opportunity to exercise the option as per the terms and conditions of the contract. The options are categorised on the basis of Time To Expiry as follows:

Very short > less than 5 days; Short >5-10 days; Medium> 10-20 days; Long>more than 20 days

The summary of data screening statistics for the Nifty options is given in Table 1

Table 1 : Data Screening Statistics for the Nifty Options

Total Call Contract Observation						61995
Criteria Observations Not satisfying Boundary conditions						3816
Observations with Less than 50 Contracts being traded						10840
Observations with very low settlement price(i.e. less than Rs.1.00)						7968
Observations for which No convergence is achieved for BSIV						5701
Rejected Data						28325
Rejected Data (in %)						45.69
Remaining Data Points considered for Analysis						33670
Sampled Data (in %)						54.31
	DOT	MOT	MAT	MIT	MDITM	
Very Short	389	515	2161	204	12	3270
Short	278	892	3897	1025	131	6223
Medium	441	2363	8457	3446	667	15374
Long	395	1638	3864	2215	691	8803
Total	1503	5408	18379	6890	1500	33670

(source: Original calculation)

IMPLIED VOLATILITY AND VOLATILITY SMILE

The volatility which is obtained by inversing the Black Scholes Option Pricing Model is known as the Implied volatility. The Black Scholes Implied volatility is the unique parameter for which the Black Scholes formula recovers out of the price of that option. So, the Black Scholes Option Pricing formula is given as below

$$C = S * N(d_1) - X * e^{-r\tau} * N(d_2) \quad \text{......(1)}$$

$$\text{Where } d_1 = \frac{\left[\ln\left(\frac{S}{X}\right)+\left(r+\frac{\sigma^2}{2}\right)*\tau\right]}{\sigma} * \sqrt{\tau} \text{ and } d_2 = \frac{\left[\ln\left(\frac{S}{X}\right)+\left(r-\frac{\sigma^2}{2}\right)*\tau\right]}{\sigma} * \sqrt{\tau} = d_1 - \sigma * \sqrt{\tau} \quad \text{......(1.1)}$$

Where 'C' is call option price, 'S' is price of underlying asset, 'X' is strike price of option, 'r' is rate of interest, 'τ' is time to expiration, 'σ' is implied volatility, 'N' standard normal distribution with mean=0 and standard deviation=1, '*ln*' is the natural logarithm. Poteshman (2000) points out that the value of underlying asset i.e. stock can be equated as $S = S e^{-\delta\tau}$, where 'S' is the value of underlying asset i.e. Nifty, δ is the dividend paid by Index, τ is the time to expiry. He suggested that the ex-post rate of dividends may not match the ex-ante expectation at the time of option is priced. He suggested that Spot-Futures parity can be used to determine $S = S e^{-\delta\tau}$ as from futures data as $S e^{-\delta\tau} = F e^{-r\tau}$, where F is the Futures value on the same date and time to expiry as that of Nifty option, r is the continuously compounded interest rate (MIBOR).

VOLATILITY SMILE MODEL SPECIFICATIONS

The Implied volatility is not observed as a constant parameter across different options as assumed in the Black Scholes Option Pricing Model. The implied volatility when observed across different levels of moneyness and time to expiry showcases the phenomenon of volatility smile. The study tries to identify the determinants of implied volatility smile by employing the different specifications (Engstrom, 2001), Dumas et al(1998), Pena et al (1999)).

Model 1: It assumes a linear relationship between implied volatility and moneyness. In other words, it assumes the existence of 'sneer' or a 'smile'.

$$\sigma = \alpha_o + \beta_1\left(\ln\left(\frac{X}{S}\right)\right) + \varepsilon \quad \text{......(2)}$$

One drawback of this specification of implied volatility function is that whenever the extreme moneyness is linearly related to volatility, then the volatility may become negative.

Model 2: It tries to include a quadratic term to capture the curvature of smile of volatility.

$$\sigma = \alpha_o + \beta_1\left(\ln\left(\frac{X}{S}\right)\right) + \beta_2 (\ln\left(\frac{X}{S}\right))^2 + \varepsilon \quad \text{......(3)}$$

Model 3: It tries to capture the effect of time effects by including time to expiration parameter, linear and cross terms with both time and moneyness. It also relates the shape of smile to the time left to expiry.

$$\sigma = \alpha_o + \beta_1 \left(\ln\left(\frac{X}{S}\right)\right) + \beta_2 (\ln\left(\frac{X}{S}\right))^2 + \beta_3 (\tau) + \beta_4 (\ln\left(\frac{X}{S}\right)(\tau) + \varepsilon \text{ ....(4)}$$

Model 4: It adds the quadratic term to the time to expiry in the previous model.

$$\sigma = \alpha_o + \beta_1 \left(\ln\left(\frac{X}{S}\right)\right) + \beta_2 (\ln\left(\frac{X}{S}\right))^2 + \beta_3 (\tau) + \beta_4 \left(\ln\left(\frac{X}{S}\right)\right)(\tau) + \beta_5 (\tau)^2 + \varepsilon \text{ ......(5)}$$

Model 5: It assumes that the volatility function is linear on the degree of moneyness, but a quadratic term is necessary to capture some degree of curvature in the right hand side of the function.

$$\sigma = \alpha_o + \beta_6 * U + \beta_9 * D^2 + \varepsilon \text{ ....(6)}$$

Where $U = \ln\left(\frac{X}{S}\right) if \ln\left(\frac{X}{S}\right) < 0; U = 0 if \ln\left(\frac{X}{S}\right) > 0$; and $D = ln\left(\frac{X}{S}\right) if ln\left(\frac{X}{S}\right) \geq 0; D = 0 if ln\left(\frac{X}{S}\right) < 0$

Model 6: $\sigma = \alpha_o + \beta_6 * U + \beta_2 (\ln\left(\frac{X}{S}\right))^2 + \varepsilon$ ....(7)

It assumes that volatility function is linear on the degree of moneyness till the options are at the money and curvature is due to all the options whether in the money or out of money.

Model 7: $\sigma = \alpha_o + \beta_7 * U^2 + \beta_8 * D + \varepsilon$ ....(8)

It assumes that curvature to the implied volatility is brought only by options which are away from at the moneyness level and ATM options bring linearity in the volatility function.

Model 8: $\sigma = \alpha_o + \beta_3 (\tau) + \beta_5 (\tau)^2 + \beta_6 * U + \beta_9 * D^2 + \varepsilon$ ....(9)

This model adds the quadratic time to expiry to the model 6.

Model 9: $\sigma = \alpha_o + \beta_{10} \frac{(\ln(\frac{X}{S}))}{\sqrt{\tau}} + \beta_{11} (\frac{\ln(\frac{X}{S})}{\sqrt{\tau}})^2$ ....(10)

This model employs the quadratic function of moneyness. Rosenberg (2000) used a definition of moneyness in terms of standard deviation units referred to as standardised moneyness in his dynamic implied volatility functions.

Model 10: Misra, Kannan (2006) gave the following model for modelling implied volatility surface which tries to explain the determinants of implied volatility surface.

$$\sigma_{i\tau,t} = \alpha_1 + \omega\left|\frac{(S_t - X_{i\tau,t})}{S_t}\right| + \gamma D_1 + \delta\tau + \mu\left|\frac{(S_t - X_{i\tau,t})}{S_t}\right| * \tau + \theta\, NOC_{i\tau,t} + \varepsilon \quad \text{.............(11)}$$

Where

$\sigma_{i\tau,t}$ is the implied volatility of an option with the exercise price of '$X_{i\tau,t}$' and time to maturity τ on trading day t, S_t is the value of underlying asset, $X_{i\tau,t}$ is the value of ith exercise price with time to maturity τ available for trading on day t, $\frac{(S_t - X_{i\tau,t})}{S_t}$ represents the extent to which the option is in the money or out of the money, τ is the time to maturity of the option with exercise price of X_i on the day t, D_1 is the dummy variable whose value is determined as follows

$$D_1 = \begin{cases} 0, if\ (S_t - X_{i\tau,t}) < 0 \\ 1, if\ ((S_t - X_{i\tau,t}) > 0 \end{cases} \begin{Bmatrix} (out\ of\ money) \\ (in\ the\ money) \end{Bmatrix} \text{.............(12)}$$

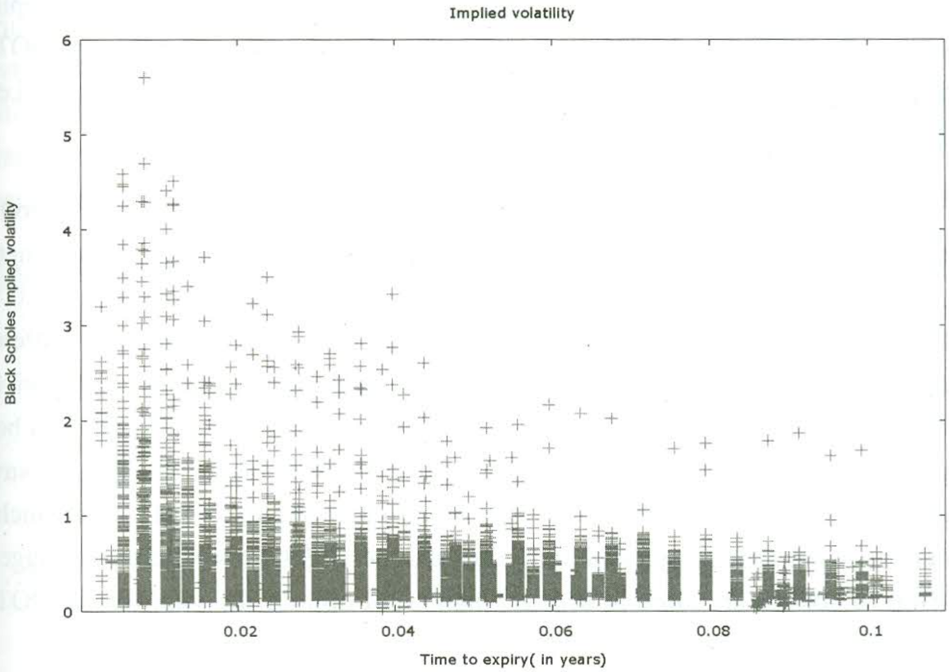
$NOC_{i\tau,t}$ is the number of contracts of options being traded on day t with exercise price $X_{i\tau,t}$

ε is random disturbance term

EMPIRICAL RESULTS AND DISCUSSION

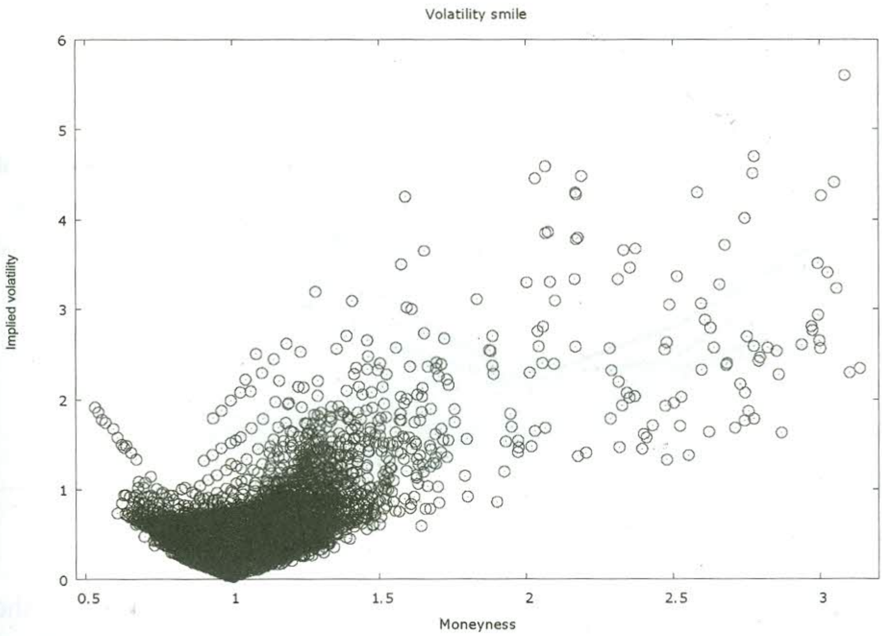
Figure 1 shows the Implied volatilities obtained across different time to expiry. Implied volatilities for each observation is calculated through equation 1 by employing Netwon Raphson method. It can be observed that the implied volatility declines with gradual increase in time to expiry. Figure 2 showcases the implied volatilities across different levels of moneyness (η) i.e. ATM, DITM, ITM, DOTM & OTM. It can be observed that the Implied volatility is lowest for the ATM options, and it increases as the options move away from ATM level of moneyness. The smile of volatility can be clearly observed in the Figure 2. The smile phenomenon is the evidence of the information asymmetry in the options market.

Figure 1: Implied volatilities across varying time to expiry



(Source: Original Calculation)

Figure 2: The volatility smile

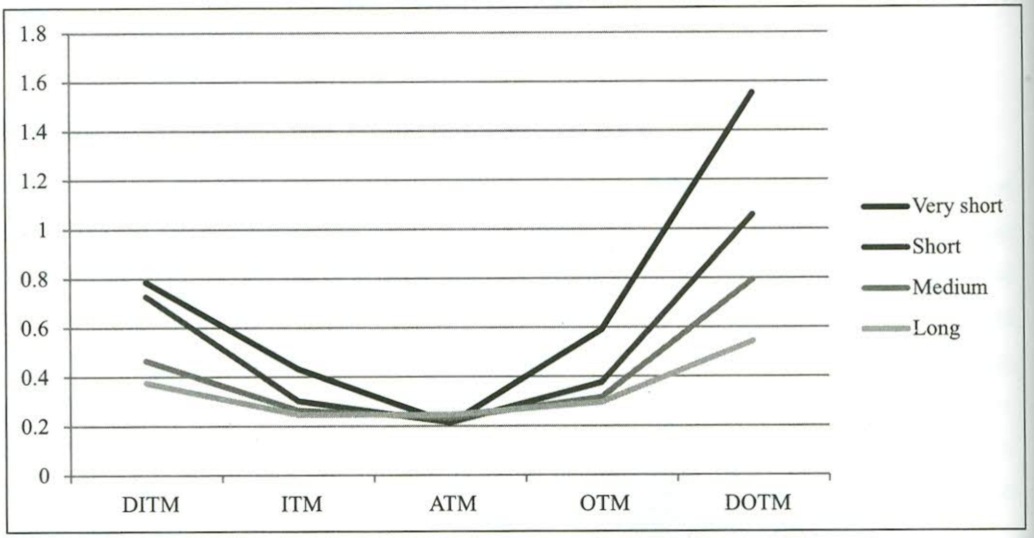


(Source: Original Calculation)

It can be observed that the average implied volatilities for DOTM, DITM options for both short and very short time to expiry have the comparatively greater implied volatilities than OTM and ITM options. The average implied volatility for DOTM options with very short time to expiry have the highest level of implied volatility i.e. it gives the peak to the implied volatility.

It is quite evident that the options which are either DOTM or DITM with very short or short time to expiry have greater amount of informational asymmetry. A symmetric volatility smile can be observed at all levels of time to expiry if we consider only ITM, ATM and OTM options. Figure 3 presents the U shape of volatility smile across different periods of time to expiry. It has been observed that if the time to expiry is comparatively shorter than the options tend to have the wry smile shape. This pattern has also been referred as 'the options are dying smiling' (Engstrom (2001)). Along with this, the smile at all levels of moneyness confirms to be more pronounced for DOTM level which is similar to what has been suggested by Duque & Lopes,1999. This pattern also suggests that DOTM options are more in demand than OTM options. In other words, DOTM options are more expensive than OTM options.

Figure 3: Volatility smiles across different time to expiry



(Source: Original Calculation)

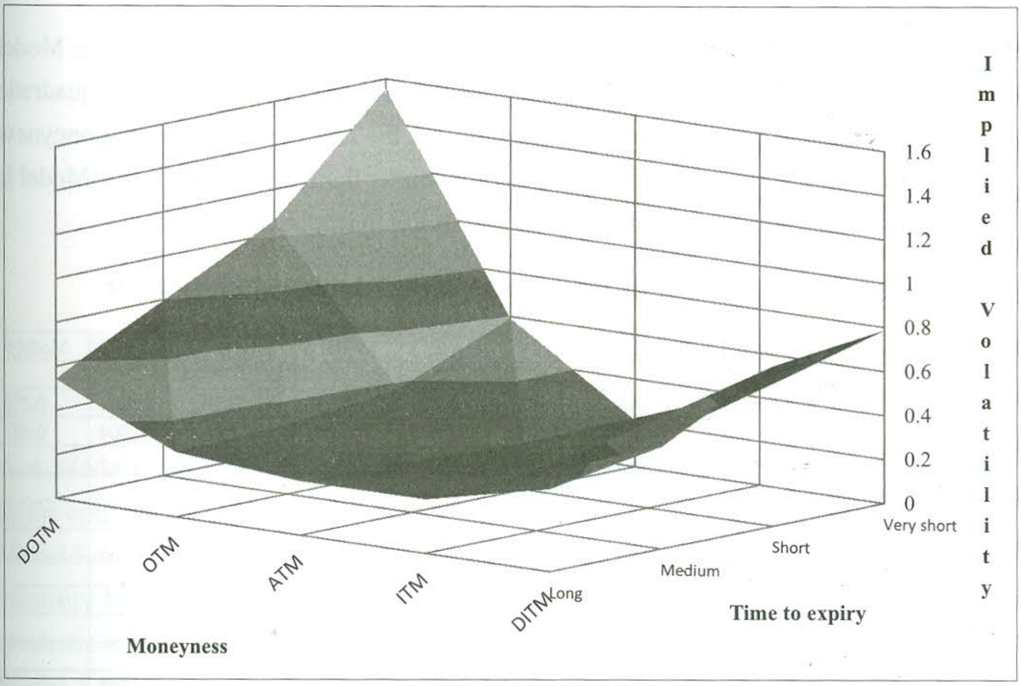
Table 2 presents the results of the Implied volatility averaged across the co-ordinates of time to expiry and moneyness. The surface generated via these co-ordinates is shown in the Figure 4 . It clearly indicates the volatility surface structure for the Indian options market. It is an indication that the informational asymmetry increases along the edges of moneyness and time to expiry. This initial attempt of defining the implied volatility along the dimensions of moneyness and time to expiry confirms the presence of the widely discussed phenomenon of volatility smile for Indian stock market.

Table 2: Average Implied Volatilities across different levels of Moneyness and Time to Expiry

	Very short	Short	Medium	Long
DOTM	1.554139	1.05765	0.794611	0.542844
OTM	0.590271	0.376701	0.315574	0.295D998
ATM	0.211469	0.213642	0.233052	0.245655
ITM	0.4324	0.302677	0.264247	0.246668
DITM	0.7857	0.727235	0.465464	0.375807

(Original Calculation)

Figure 4: The volatility surface



(Source: Original Calculation)

In the next step, we have tried to analyse the variables which can define the shape of the volatility smile; and have tried to find answers to the questions like 'What gives the smile its curve?'. For this the models proposed have been analysed and the results are shown in Tables 3 to 5.Out of the given models, model 8 has outperformed the rest of the all. Model 4 , 3 and 5 have also performed well. On the other hand, Model 1 and 9 have underperformed the most.

Model 1 (referring to equation no. 2) fails to establish the linear relationship between volatility and moneyness. One possible reason for the poor performance of model 1 can be inclusion of varying levels of moneyness which ultimately rejects the assumption of constant volatility across different levels of moneyness. The curvature posed by the volatility smile can be explained by inclusion of curvature in the model. It is clearly evident that a mere inclusion of quadratic term to the definition of volatility smile improves the performance of model significantly. Model 2 (referring to equation 3) can be rightly said as an explanatory model for volatility smile as it explains 53% approximately of the variability of volatility in relation to moneyness. This model has also been considered as best model for explaining the volatility smile by Pena et. al (1999). But the results improve on inclusion of time dimension in the implied volatility function as carried out in Model 3 and 4 (referring to equation no. 4 and 5 respectively).

Basically Model 3 and 4 try to explain the implied volatility by volatility surface. Model 4 captures the curvature of implied volatility function rightly by including the quadratic terms of moneyness and time to expiry, along with the interaction between moneyness and time to expiry. Due to this reason, it performs better than initial models (i.e. Model 1, 2 and 3) and explains around 59.8 % of the variability in Implied volatility.

Table 3: Results of different models explaining volatility smile

	Model1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
	0.001								
	0.297	-0.2673	0.3056	0.344	0.197	0.231	0.254	0.3007	0.275
		0.001	0.001	0.003	0.0009	0.001	0.0011	0.004	0.001
	-1.227	-.4656	-1.5843	-1.548		1.597			
	0.046	0.02	0.19	0.019		0.137			
		2.8766	2.53	2.541				-3.62	
		0.128	0.02	0.02				.220	
			-0.9669	-3.058					
			0.04	0.154					
			25.77	22.336					
			0.36	1.59					
								22.148	
								2.048	

					-2.44	-1.5057		-2.415	
					0.010	0.047		0.046	
							3.369		
							0.0192		
							0.632		
							0.0191		
					5.457			5.758	
					0.07			0.1406	
									-0.107
									0.006
									0.0435
									0.003
Adjusted R2	0.26670	0.5245	0.596	0.598	0.6081	0.584	0.479	0.62817	0.62059
Std Error	0.2110	0.1699	0.1222	0.156	0.154	0.158	0.177	0.1502	0.151822
F-statistic	-696.824	635.17	12429.67	10041.17	26121.49	1491.851	15480.27	916.836	1195.518
p-value	0.0000	0.0000	0.0000	0.0000	0.00000	0.00000	0.0000	0.0000	0.0000

(Source: Original Calcuation)

Model 5, 6 and 7 (referring to equation no. 6,7 and 8) focus on deciding which level of moneyness actually adds curvature to the volatility smile. Model 5 considers that linearity is added by OTM options while the curvature is added only by ATM options. On the other hand, Model 7 assumes the vice versa of Model 5.

Table 4: Results of the Model 10

Model 10			
	Coefficient	Std. Error	p-value
	0.227	0.0023	0.0000
	3.95743	0.09	0.0000
	4.799	0.078	0.0000
	-1.751	0.05	0.0000
	-21.4335	1.37	0.0000
	0.00000	0.0000	0.0000
Adjusted R2	0.64202	S.E	0.147
F-statistic	2628.096	p-value	0.0000

And, Model 6 considers that linearity is due to OTM options and curvature is brought by all the options whether ITM, ATM or OTM. It can be seen from the results that the out of Model 5, 6 and 7, Model 5 explains approximately 61 % of the variability of the implied volatility function. The results presented by Model 5 get further improved when the quadratic term of time to expiry is also added in the model, which leads to the model 8. Model 8 (referring to equation no. 9) explains the volatility smile best in terms of moneyness and the time to expiry. Looking at the coefficients it can be seen that β_5 has the most significant role in explaining the curve of volatility smile. The positive and significant value of β_5 clearly indicates that nearer the time to expiry of an option, the higher the volatility of the option and deeper would be the smile. The coefficientβ_7 which explains the quadratic moneyness level of at the money options is also significant. Implied volatility function's explanation would be incomplete by just considering the dimensions of volatility smile (i.e. moneyness and implied volatility) so inclusion of term defining interaction between moneyness and time to expiry is evident in the implied volatility function, which is significantly supported by the coefficient β_4.

The results of Model 10 (referring to equation no. 11) are shown in Table 4. ω is found as positive and significant which indicates that the OTM and ITM options have higher implied volatility than ATM options. γ indicates that OTM, ITM have higher volatility than ATM options. δ is just exceeding one which implies that out of money options which have nearer time to expiry have higher volatility than options with far than time to maturity. μ is comparatively having the highest negative value which indicates that short term options, i.e. OTM have the higher volatility than long term options which are ITM. θ is almost zero and this negligible value which indicates that there is little difference caused in explanation of implied volatility by the options which are more liquid than the options which are less liquid. The given model is able to capture almost 60% of the variability of implied volatility.

In sample Performance comparison of models: The in-sample performance of the models can be judged on the basis of results shown in table 5. It is clearly shown that Model 8 which is followed by Model 5 is the one which get the best fit of in-sample forecasting performance. This is in coherence with the results found previously.

Table 5: Insample forecasting errors for different models

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9	Model10
ME	-7.81E-18	-8.82E-18	-8.01E-18	-6.68E-18	3.87E-20	7.58E-19	-2.23E-18	-2.56E-18	-7.44E-18	-1.25E-17
M SE	0.041511	0.026499	0.023308	0.023243	0.023459	0.024314	0.028129	0.021862	0.039926	0.020238
RMSE	0.20374	0.16278	0.15267	0.15246	0.15316	0.15593	0.16772	0.14786	0.19981	0.14226
MAE	0.12502	0.098025	0.098427	0.098584	0.094703	0.095521	0.099117	0.094326	0.11105	0.089492
MPE	-22.198	-17.777	-17.175	-17.306	-12.809	-15.822	-18.187	-12.197	-21.003	-10.157
MAPE	43.989	34.802	35.575	35.771	31.889	33.274	35.071	32.339	38.646	29.94

CONCLUDING OBSERVATIONS

The non constant nature of implied volatility across different strike prices (X) for the same underlying asset (S) and time to expiry (τ) gives rise to non-linear pattern of implied volatilities across τ and η give rise to the phenomenon of 'Volatility Smile' which was first traced on S&P 500 index options before 1987 stock market crash across the world.

In the present study, the pattern of implied volatility observed across the dimensions of 'τ' and 'η' indicated the evidence of presence of volatility smile in Indian options market. It also indicated the presence of informational asymmetry in the options market especially signalled by DITM and DOTM options. The study has also attempted to find answer to the question 'What gives the smile its shape?'. It has been found that out of all the models, Model 8 (which includes linear as well as quadratic terms of η and τ in specifying the volatility) has performed best as it included a quadratic term of time to expire and moneyness which leads to a better capture of the curvature in the implied volatility pattern. The time to expiry and extent of moneyness are not only significant parameters but their interaction with each other is also an important factor which gives the curve to volatility smile. The phenomenon of volatility smile cannot be explained by employing the linear relationships, rather, the quadratic functions significantly explain the implied volatility functions. The study would help traders in analysing the implied volatility and the price of call options in order to determine the expensiveness of the options.

Past studies on volatility smile have indicated that the shape of the volatility smile varies from market to market. Shastri and Wethyavivorm (1987), Pena et. al (1999) and Engstrom (2002) found the shape of volatility smile to be U-shaped in Philadelphia market, symmetric in Spanish market, negatively sloped in Italian market, and positively sloped in case of Swedish market respectively. However, the findings of the study have indicated the negative slope of the volatility smile in case of Indian market.

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Shri Ram Centre

For Industrial Relations, Human Resources, Economic & Social Development,

Unit No. 1078 (F/F), Central Square, Plaza-II,
M.L. Khurana Marg, (Barahindu Rao), Delhi-110006
Ph. 011- 23635816, Fax: 23635815
E-mail: srcpublications@gmail.com Website:www.srcirhr.com